Experimental Prediction of Nusselt Number and Coolant Heat Transfer Coefficient in Compact Heat Exchanger Performed with $\varepsilon$-NTU Method

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Abstract  
In this Study, radiator performance for passenger car has been studied experimentally in a wide range of operating conditions. Experimental prediction of Nusselt number and heat transfer coefficient for coolants in radiator tubes are also performed with $\varepsilon$-NTU method. The total effectiveness coefficient of radiator and heat transfer coefficient in air side is calculated via trial and error method considering experimental data. The Colburn factor and pressure drop are also estimated for this heat exchanger. Examples of application demonstrate the practical usefulness of this method to provide empirical data which can be used during the design stage.

Keywords: Compact Heat Exchanger, Heat Transfer Coefficient, Nusselt Number, $\varepsilon$-NTU Method
Introduction:

The new emission regulations (Euro4 and EPA02) will require higher performance and efficiency of a truck cooling system. The truck cooling system could be divided into two parts, heat exchangers, (radiator, charge air cooler, oil cooler, etc) and airflow management components (fan, fan clutch and drive, shroud). The engine cooling radiators are basic components on which the correct engine operation and its performance depend. Predicting the radiator behaviour is an important issue that helps the radiator designers to project them correctly once they know the real boundary condition. Due to limited space at the front of the engine, the size of the heat exchangers is restricted and cannot be essentially increased. Therefore, to meet these higher cooling demands, it is very important to work with optimization of other components of the cooling system such as fan and fan shroud. Nevertheless, the optimization of the truck cooling system involves not only in optimization of each single component but even in analysis of interaction between them. It is also fundamental to have a good knowledge about the efficiency of the cooling system at an early stage of truck development in order to save time and costs.

Considerable success has been reported about improving heat transfer effectiveness, de-clutching or modulating the cooling fan [1], improving the efficiency of the ram path [2,3] and the management of air the inside the engine component [3,4]. Ram air effect on the airside cooling system performance has been reported by Schaub and Charlies [5]. There are several related investigations about heat transfer (Colburn factor) and friction correlation for compact louvered fin-and-tube heat exchangers in [6,7].

Test equipments

In the experimental tests, the automobile is connected to Eddy current dynamometer and all actual moving conditions are imposed on it. As shown in Fig.1, two axial fans with associated channels are employed in order to model the moving car on road. The air flow rate is controlled by the moving gate installed at the inlet of the channel. All tests are performed based on SAE J2082 JUN92 standard.

In this research, the Pitot tube and wane anemometer are used to measure air velocity. K type thermocouples and thermometers connected to vane anemometer were used for temperature measurement. Coolant flow rate was obtained by rotameters. The multimeter was employed to estimate the electro power consumption. The Diagnostic system was also employed to determine the motor parameters that were controlled by ECU.

![Fig. 1. Test bench](image)

Table 1 has shown engine type and the radiator specification that are used in this study.

<table>
<thead>
<tr>
<th>Vehicle Name</th>
<th>Pride</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine Type</td>
<td>4 Stroke, 1300CC</td>
</tr>
<tr>
<td>Fin and Tube Material</td>
<td>Al</td>
</tr>
<tr>
<td>Width(mm)</td>
<td>382.4</td>
</tr>
<tr>
<td>Dimensional Depth(mm)</td>
<td>27</td>
</tr>
<tr>
<td>Height(mm)</td>
<td>320</td>
</tr>
<tr>
<td>Fin Type</td>
<td>Corrugated</td>
</tr>
<tr>
<td>Fin Pitch(mm)</td>
<td>1.25</td>
</tr>
<tr>
<td>Fin Total Area(m2)</td>
<td>4.7</td>
</tr>
<tr>
<td>Tube Total Area(m2)</td>
<td>0.59</td>
</tr>
<tr>
<td>Heat Transfer Total Area(m2)</td>
<td>5.29</td>
</tr>
<tr>
<td>Radiator Weight(kg)</td>
<td>2.8</td>
</tr>
<tr>
<td>Cap operation Pressure(kg/cm2)</td>
<td>0.9 0.15</td>
</tr>
<tr>
<td>Radiator Volume(lit)</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Flow and temperature maldistribution

In calculating heat exchanger performance, it is normally assumed that flow is uniform in both rate and temperature distribution. In practice, neither is likely to be true. While the effects might be relatively small on the coolant side on the airside, the effects can be very large. The causes of this maldistribution include [8]:

- Heat load and flow resistance from air-conditioning and auxiliary heat exchangers placed in front of the radiator.
- Non-uniform inlet conditions caused by the radiator grille.
- Inherent mismatch between the annular flow of the fan and the rectangular heat exchanger. (The fan shroud will never be deep enough to allow full transition).
- The proximity of objects placed downstream of the fan, cause asymmetric fan airflow.
- Hot air recirculation caused by low pressure between the grille and the heat exchangers.

Fig. 2 and 3 show histograms of radiator airflow maldistribution obtained from CFD analysis of a Ford F350 Vehicle, with ducting and seals removed.

Chiu [10] have suggested that radiator heat transfer effectiveness "Deteriorates due to two-dimensional flow non-uniformity on both the air and coolant sides".

Therefore, temperature distributions as well as the non-uniformity of the cooling airflow across the radiator were measured.

Radiator performance

Two common methods exist for expressing the heat transfer characteristics of a given heat exchanger surface geometry. These are known as LMTD approach and the \( \varepsilon \)-NTU approach. The \( \varepsilon \)-NTU approach of a heat exchanger can be determined by examining the heat loss and heat gain that takes place between its working fluids:

The heat lost by the coolant can be expressed as:

\[
Q_h = \dot{m}_h C_{ph} (T_h - T_{hi})
\]  

Heat gained by the air can be expressed as:

\[
Q_c = \dot{m}_c C_{pc} (T_c - T_{ci})
\]  

And the capacity rate, \( C_r \), is defined as [8]:

\[
C_r = \frac{Q_c}{C_{pc}}
\]

\[
C_r = \frac{Q_h}{C_{ph}}
\]  

The smaller one is defined as \( C_{min} \) and the larger \( C_{max} \) and \( C_r \) is the capacity ratio:

\[
C_r = \frac{C_{min}}{C_{max}}
\]  

The number of transfer units, NTU is defined as [8]:

\[
NTU = \frac{AU}{C_{min}} = \frac{1}{C_{min}} \int UdA
\]  

When the inlet temperature and flow rates are specified, the maximum heat transfer rate possible, (for an infinitely sized counter flow heat exchanger), is given by:

\[
Q_{max} = C_{min} (T_h - T_{ci})
\]  

If effectiveness, \( \varepsilon \), is defined as the ratio of actual dissipation to maximum dissipation we get [8]:

\[
\varepsilon = \frac{Q}{Q_{max}} = \frac{C_r (T_h - T_{hi})}{C_{min} (T_h - T_{ci})} = \frac{C_r (T_c - T_{ci})}{C_{min} (T_h - T_{ci})}
\]
Effectiveness can be expressed as a function of NTU, the capacity ratio and heat exchanger configuration:

$$\varepsilon = \varepsilon (NTU, C_r, Flow \, Arrangement)$$  \hspace{1cm} (8)$$

There are many equations for $\varepsilon$ that are obtained by different authors, but in real condition and for one row tube radiators, we can use equations (9) and (10):

$$\varepsilon = 1 - \exp \left(-\frac{C_r}{C_p} \left(1 - e^{-NTU} \right) \right) \hspace{1cm} (9)$$

$$\varepsilon = 1 - \exp \left[-\frac{1 - e^{-NTU/C_r}}{C_r} \right] \hspace{1cm} (10)$$

The flow of heat from the hot coolant to the cold air, can be represented by Newton's Law of cooling as shown in equation (11):

$$Q = U(A_h - T_c)$$  \hspace{1cm} (11)$$

Where the overall heat transfer coefficient, $U$, comprises three separate resistances as shown in equation (12):

$$\frac{1}{UA} = \frac{1}{U_aA_e} + R_{f} + \frac{1}{U_rA_r}$$  \hspace{1cm} (12)$$

If we assume the fin efficiency, $\eta_f$, then equation (12) can be rewritten to give:

$$\frac{1}{U_aA_e} = \frac{1}{A_fh_f} + 1 + \frac{1}{K} + \frac{1}{\eta_fA_fh_f}$$  \hspace{1cm} (13)$$

And total surface efficiency, $\eta_s$:

$$\eta_s = 1 - \frac{A_f}{A} (1 - \eta_f)$$  \hspace{1cm} (14)$$

$A$ = surface area in which $U$ is based.

For a flat tube heat exchanger it can be shown that [8]:

$$\eta_f = \frac{\tan h \left( \frac{2h_c}{\sqrt{K\delta}} \right)}{\frac{2h_c}{\sqrt{K\delta}}}$$  \hspace{1cm} (15)$$

The most commonly used relationship for laminar flow (Re<2300) is the correlation proposed by Seider and Tate in 1936 [8]:

$$Nu = 1.86 \left( Re, Pr \frac{D_s}{L} \right)^{1/2} \left( \frac{\mu_s}{\mu_c} \right)^{0.14}$$  \hspace{1cm} (16)$$

Where,

$$Nu = \frac{\sqrt{\frac{\mu_s}{\mu_c}}}{\sqrt{\frac{\mu_s}{\mu_c}}}$$  \hspace{1cm} (17)$$

Reynolds number, Re is defined as:

$$Re = \frac{\rho u D_s}{\mu}$$  \hspace{1cm} (18)$$

Prandtl number, Pr is defined as:

$$Pr = \frac{\mu C_p}{K}$$  \hspace{1cm} (19)$$

And

$$D_s = \frac{4 \times \text{Flow Area}}{\text{Wetted Perimeter}}$$  \hspace{1cm} (20)$$

Modification to the Petukhov model, by Gnielinski using experimental data has extended the correlation to include the transitional range (2300<Re<10000) where most automotive radiators operate [8]:

$$Nu = \frac{\sqrt{\frac{\mu_s}{\mu_c}}}{\sqrt{\frac{\mu_s}{\mu_c}}} \left( Re_s - 1000 \right) \frac{Pr_t}{1 + 12.7 \left( \frac{Pr}{2} \right)^{1/2} \left( \frac{Pr}{2} - 1 \right)^{1/2}}$$  \hspace{1cm} (21)$$

Where fanning friction factor, $f$ is given by [8]:

$$f = \left( 1.58 \ln Re_s - 3.28 \right)^2$$  \hspace{1cm} (22)$$

To facilitate the comparison of different heat exchanger geometries, the heat transfer coefficient is normally made into a non-dimensional number using Stanton number, $St$, or Colburn factor, $j$, and compared at constant Reynolds number, Re [8].

$$St = \frac{\text{convective heat flux}}{\text{fluid heat capacity rate}} = \frac{h_c}{\rho u C_p}$$  \hspace{1cm} (23)$$

Colburn factor is defined as [8]:

$$j = St \frac{2}{Pr^3}$$  \hspace{1cm} (24)$$

Using the following procedure, Colburn factor ($j$) can be determined:

- For best accuracy select data for a coolant flow rate where $Re$>4000.
- As the inlet and outlet temperatures and flow rates are known on both sides of the heat exchanger calculate effectiveness, could be calculated using equation (7).
- Using the appropriate $\varepsilon$-NTU relationship, NTU, and using equation (5) $UA$, could be calculated.
- Using the Gnielinski correlation of equations (21) and
(22) the coolant side Nusselt number is obtained. To calculate the heat transfer coefficient, \( h_c \),

- Substitute for \( h_c \), \( A_c \), \( t \) and \( k \) into equation (25):

\[
\frac{1}{UA} = \frac{1}{U_c A_c} = \frac{1}{A_c h_c} + \frac{t}{K} + \frac{1}{\eta_c A_c h_c}
\]

(25)

To give,

\[
\frac{1}{UA} = k + \frac{1}{\eta_c A_c h_c}
\]

(26)

Where constant \( k \) is given by:

\[
k = \frac{1}{A_c h_c} + \frac{t}{K}
\]

(27)

Substituting equation (14) into equation (27), gives equation (28).

\[
\left(1 - \frac{k}{UA}\right)^{-1} = h_c A_c \left(1 - \frac{A_c}{A} \left(1 - \eta_c \right)\right)
\]

(28)

- Using the fin efficiency relationships equations of (14) and (15), we can solve fin efficiency, \( \eta_c \) and air side heat transfer coefficient, \( h_c \). As fin efficiency is a function of heat transfer coefficient, the solution will be iterative.

- Stanton Number can, then, be determined using equation (23) and Colburn factor using equation (24).

**Air-side pressure drop in radiator**

Following Kays and London results, air-side pressure drop can be calculated from the following relationship [8]:

\[
\Delta P = \frac{G^2}{2g} \frac{1}{\rho_i} \left[ \text{entrance effect} + \text{flow acceleration} \right]
\]

\[+ \text{core friction} - \text{exit effect} \]

(29)

\[
\Delta P = \frac{G^2}{2g} \frac{1}{\rho_i} \left[ \left(K_e + 1 - \delta^2 \right) + 2 \frac{\rho}{\rho_i} \left(1 - \delta^2 \right) \right]
\]

\[+ \frac{A}{k \rho_i} \left(1 - \rho^2 - K_e \frac{\rho}{\rho_i} \right) \]

(30)

**Error and accuracy analysis**

It is obvious that any empirical study has special errors, such as application, operating, environmental, dynamic and calculating.

Type K thermocouples are more frequently used in industry. They are usually calibrated in the range 0°C to 1100°C, with expanded uncertainty of ± 1.0°C up to 1000°C.

We calibrated all thermocouples with comparison method and repeatability of all instruments is examined. To notice response time error, we read all data after reaching 98 percentage of standard value. Reproducibility of procedures is other the criteria to be confidence of experimental data, and then we almost repeated any of exams for two times. We used all indicators with 10 times resolution higher than accuracy that we have accepted.

For uncertainty analysis in equation 1, we can use the method below:

\[
R = f(x_1, x_2, ..., x_n)
\]

\[
W_R = \left[ \left( \frac{\partial R}{\partial x_1} W_1 \right)^2 + \left( \frac{\partial R}{\partial x_2} W_2 \right)^2 + ... + \left( \frac{\partial R}{\partial x_n} W_n \right)^2 \right]^{1/2}
\]

Substituting equation (14) into equation (27), gives equation (28).

\[
\frac{\partial Q}{\partial m_k} = C_{p,h} \left(T_k - T_i\right), \quad \frac{\partial Q}{\partial T_k} = m_k \left(T_k - T_i\right)
\]

For special test:

\[
m_k = 60 \pm 5\% \left( \frac{lt}{min} \right), \quad T_k = 90 \pm 1^\circ C
\]

\[
T_i = 83 \pm 1^\circ C, \quad C_{p,h} = 4200 \pm 0.3\% \left( \frac{j}{kg \cdot K} \right)
\]

\[
\Rightarrow W_1 = 0.02, \quad W_2 = 21, \quad W_3 = 1, \quad W_4 = 1
\]

**total uncertainty** = \( \frac{W_k}{Q} \times 100 = 8.8\% \)

Indeed, the total uncertainty in radiator heat loss equation is 8.8%.

**Results and discussion**

At constant engine speed, increment of air flow in front of vehicle causes that temperature of the engine compartment air is reduced. Also, air velocity growth in this cavity affects heat transfer coefficient of engine body and raises the heat loss of the engine and other solid parts of this space. Fig. 4 shows the component of energy balance at 3000 rpm and 62 km/hr.

The influence of engine speed on distribution of energy is presented in Fig. 5. Increment of engine speed decreases time of heat transfer and increases motion of cylinder gas which leads to engine heat transfer coefficient rising. Gas...
temperature increment due to engine speed, grows heat transfer of engine body and radiator. Also, it directly affects the exhaust loss and amplifies its magnitude [11].

Regarding the position of the vehicle elements especially in front of it, exhaust airflow from condenser is inlet airflow for radiator. Therefore, as shown in Fig. 6, we can find non-uniform temperature distribution of inlet airflow for radiator (at special vehicle speed, 41 (km/hr)). Also, because of the coolant flow path, we can see non-uniform temperature distribution in outlet airflow (at inlet coolant temperature, 95 (°C)). (Fig. 7)

![Fig 6: Inlet airflow Temp. Distribution for radiator at inlet coolant Temp. (95 °C)](image)

![Fig 7: Outlet airflow Temp. Distribution for radiator at vehicle speed 41 km/hr (inlet coolant Temp. is 80 °C)](image)

Fig. 4: Distribution of energy balance at 3000 rpm and 62 km/hr.

![Fig 5: Influence of engine speed on distribution of energy](image)

Fig. 5: Influence of engine speed on distribution of energy

As shown in Fig. 8, in constant vehicle speed and considering gear conversion ratio, increment the number of gear causes reduction of heat loss. Indeed, increment the number of the gear is linked together with the reduction of engine rpm and also the coolant flow rate and regarding equation (1), the heat loss will be decreased.

As shown in Fig. 9, heat transfer rate has been shown via coolant Reynolds number at different gears and maximum load. At special coolant Reynolds number, increasing the car velocity leads to an increase in the airflow rate through radiator and with reference to equation (1), the heat transfer rate increase. On the other hand, increasing the water
pump speed is the main reason for increasing the heat transfer rate because it affects coolant flow rate; therefore, the heat transfer rate.

The value of Nusselt number versus coolant Reynolds number has been shown in Fig. 10. The coolant heat transfer coefficient has been depicted versus Reynolds number in Fig. 11. As shown, increasing the Reynolds number leads to increasing the coolant heat transfer coefficient that strongly affects the total heat transfer rate of radiator. The value of Nusselt number has been shown in Fig. 12, versus the inlet air velocity at different gears. As is shown in Fig. 12, airflow distribution and inlet air velocity to radiator have significant effect on the heat transfer rate.

\[ j = 0.5457 \, \text{Re}^{-0.277} \]  
(32)

**Fig. 11.** Coolant heat transfer coefficient versus coolant Reynolds number

**Fig. 12.** Nusselt number versus vehicle speed in different gears

**Fig. 13.** Colburn factor via Reynolds number for radiator in actual condition installed in automobile

Radiator Colburn coefficient can be obtained as a function of Reynolds number as follows:

\[ j = a \, \text{Re}^b \]  
(31)

\( a \) and \( b \) are coefficients obtained from experimental results (Fig. 13). Equation (31) is a linear behavior in logarithmic plane. The radiator Colburn coefficient for available test case is:

With reference to equation (29), the air pressure drop is affected by core friction, flow acceleration, inlet and outlet effects, inlet air density and inlet air velocity. The distribution of air pressure drop through radiator has been shown versus vehicle speed in Fig. 14. Increasing the pressure...
drop through radiator is reasonable because increasing the air velocity through radiator makes the airflow more turbulent between the radiator fins and, therefore, the influence of core friction is more apparent.

Air heat transfer coefficient is affected by many parameters because of complex flow behaviour between the radiator fins. One of the important parameters is the air velocity through radiator or car velocity. In Fig. 15, the influence of car velocity on air heat transfer coefficient has been shown. As shown, the heat transfer coefficient is related to many parameters and, therefore, it has a different trend at different speeds but increasing the air velocity leads to an increase in the heat transfer coefficient at all speeds.

The total effectiveness coefficient of radiator has been shown versus the car velocity in Fig. 17 at different speeds. As is shown when the car speed increases from a specified level, the effectiveness coefficient does not depend on car velocity and converges to 0.7.

The radiator based on its dependency to airflow distribution approaches to maximum capacity when the speed increases. In a similar diagram, the total effectiveness coefficient of radiator can be shown versus the vehicle's speed at different coolant flow rates (Fig. 18). This diagram is a general state incorporating the total radiator performance at different flow rates and can be used in future designing.

As shown in Fig. 18, increasing the vehicle speed at special coolant flow rate (constant speed of engine), leads to a decrease in the total effectiveness coefficient with references to equation (3) and (7), minimum total heat capacity is related to airside, therefore, in equation (7), with increasing the vehicle speed, the mass flow rate through radiator increases and in the constant coolant flow rate, the total effectiveness coefficient decreases.
Conclusions

In this study, radiator performance for passenger car has been studied experimentally in a wide range of operation conditions. The total effectiveness coefficient of radiator and heat transfer coefficient in air side is calculated via trial and error method considering the experimental data. The Colburn factor and pressure drop were estimated for this heat exchanger. The dependency of heat transfer rate to airflow distribution was determined from the difference between real conditions in the wind tunnel and obtained results from the experiment. Heat transfer results in wind tunnel improved about 50 percent due to better air distribution in radiator, compared with actual conditions in an automobile.

Nomenclature

\begin{itemize}
  \item $A$: Total heat transfer area on $C_{\text{min}}$ side (m$^2$)
  \item $A_c$: Minimum free flow area (cross-sectional area) (m$^2$)
  \item $C$: Cold fluid (air)
  \item $C_p$: Specific heat capacity (kJ/kg.K)
  \item $G$: Mass velocity (kg/s)
  \item $g$: Gravitational constant (m/s$^2$)
  \item $h$: Hot fluid (coolant), Heat transfer coefficient (W/m$^2$.K).
  \item $i$: Inlet
  \item $k_c$: Entrance loss coefficient
  \item $k_e$: Exit loss coefficient
  \item $k$: Thermal conductivity of wall (W/m.K)
  \item $m$: Mass flow rate (kg/s)
  \item $o$: Outlet
  \item $Q_e$: Shaft output
  \item $Q_c$: Cooling loss
  \item $Q_{ex}$: Exhaust loss
  \item $Q_e$: Engine body heat loss
  \item $Q_{exp}$: Heat of the exhaust gas flow
  \item $Q_{unb}$: Heat of the unburned components
  \item $T$: Overall heat transfer (kW)
  \item $t$: Temperature (°C)
  \item $w$: Wall thickness (m)
  \item $U$: Overall heat transfer coefficient (W/m$^2$.K)
  \item $\delta$: Fin thickness
  \item $\sigma$: Ratio of free flow area to frontal area
  \item $\varepsilon$: Total effectiveness coefficient
\end{itemize}

References