Steady-state modeling of a turbocharger in gasoline engines

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ABSTRACT

The gas flow estimation is crucial for the proper operation and monitoring of turbocharged (TC) engines with a torque structured engine control unit (ECU). This paper presents mean value models developed for modeling gas flow over the compressor, the turbine and the wastegate (WG) in a TC gasoline engine not equipped with a hot film air-mass flow meter (HFM). The turbine flow was modeled by an isentropic nozzle with an effective pressure ratio. A physically sensible model was developed for estimating the WG flow and its position which estimates them over a wide range of the engine operating conditions. Finally, a novel model for estimating the compressor air flow was proposed, using coupling between powers generated by the turbine and used by the compressor. All models are simple and accurate enough to be utilized for engine real-time control applications. Experimental results from a modern gasoline TC engine were used to validate all the models. Prediction results from models were in very good agreement with the experimental measurements.

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1) Introduction

Engine downsizing using turbochargers in gasoline engines is a common solution to reducing fuel consumption and emission for automotive manufacturers [1]. In fact, displacement reduced turbocharged engines regenerate thermal power from exhaust gas either by a wastegate controlled or a variable geometry turbine. The turbochargers controlled by the wastegate (WG) benefit low cost and high durability in the harsh thermal environment of the exhaust manifold in the current automotive market [2].

When a turbocharger is added to a naturally aspirated engine, different strategies should be included in the engine control unit (ECU). These strategies are either to control the turbocharger [3-4] or to monitor its position and flow. As a new method, the coupling model is used to estimate parameters required to control the turbine actual pressure ratio. Then, the turbine flow is calculated with a new approach from the path from the compressor to the throttle is controllable by the wastegate (WG) benefit low cost and high durability in the harsh thermal environment of the exhaust manifold in the current automotive market [2].

After the introduction, the experimental setup used for validation in all parts of this paper is described in section 2. Section 3 addresses modeling of the turbine gas flow. The WG model to estimate its flow and position in different engine operating points is explained in section 4. Finally the model for detection of the compressor air flow is outlined in section 5.

2) Experimental setup

A 4-cylinder port fuel injected gasoline turbocharged engine is used for validation of all models in this work. As shown in Figure 1, the engine is equipped to a turbocharger with a radial turbine and a centrifugal compressor.

![Figure 1: Schematic of the engine test bed](image)

The turbine is controlled by a wastegate which allows bypassing the exhaust gas from the turbine. Opening the WG reduces exhaust back pressure (which improves the fuel economy) in the operation points where the engine does not require high boost pressure. The WG position is controlled by a pneumatic actuator. The pressure inside the actuator is controlled by an electronic solenoid valve which combines pressures downstream and upstream the compressor. Figure 1 also illustrates the engine measurement setup instrumented to a range of different sensors. A proximity sensor is used to measure the rotor speed with ±500 rpm resolution. Temperatures are measured using K-type thermocouples with an accuracy of ±2 °C and pressures are measured using piezoresistive pressure transmitters with ±0.1 kPa accuracy. The fuel mass flow is measured using a temperature-controlled fuel mass flow meter with a measurement accuracy of 0.12% and maximal measuring frequency of 20 Hz. The measured fuel in the test cell is used along with the measured engine air-fuel ratio.
to compute the air-charge required to validate flow models in this work since the engine is not equipped with an HFM. A hall-effect sensor with -40 to +150°C working temperature is integrated to the engine setup for measuring the WG displacement. The hall sensor is connected to the WG actuator using a thermal isolating pad. Further specifications of the engine are presented in Table 1.

### Table 1: Engine specifications

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>1.7</td>
<td>lit</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>9.9</td>
<td>-</td>
</tr>
<tr>
<td>No. of cylinders</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Bore x Stroke</td>
<td>78.6 × 85</td>
<td>mm</td>
</tr>
<tr>
<td>Max. torque</td>
<td>215 @ 2200-4800 rpm</td>
<td>N.m</td>
</tr>
<tr>
<td>Max. power</td>
<td>110 @ 5500 rpm</td>
<td>kW</td>
</tr>
<tr>
<td>Max. accessible WG</td>
<td>12</td>
<td>mm</td>
</tr>
</tbody>
</table>

3) **Turbine flow model**

Turbomachine performance characteristics such as mass flow and efficiency are specified by terms of pressure ratio and rotating speed. For these machines, the effect of inlet conditions are canceled by defining corrected mass flow and corrected speed. Therefore, for a machine of a specified size that handles a single gas, the dimensional analysis to compressible fluids suggest the following [17]:

\[
\dot{m}_{\text{corr}} = f(Pr, \omega_{\text{corr}}) \tag{1}
\]

where \(\omega_{\text{corr}} = \sqrt{T_{\text{us}}}\) is the corrected rotating speed of the machine shaft and the corrected mass flow is defined as \(\dot{m}_{\text{corr}} = \dot{m} \sqrt{T_{\text{us}} / P_{\text{us}}}\) for a turbine and \(\dot{m}_{\text{corr}} = \dot{m} \sqrt{T_{\text{ds}} / P_{\text{ds}}}\) for a compressor. The pressure ratio, \(Pr\), can be either the upstream to downstream turbomachine pressure ratio or the reverse.

Radial turbines have rather weak dependency upon the corrected speed [17]; and the pressure ratio term is the most significant variable in Equation 1. Figure 2 compares the measured corrected flow over the radial turbine with that of a conventional nozzle with a unit discharge area. The test procedure for measuring the presented data is illustrated in Figure 2(a). The flow over the turbine is measured by closing the wastegate completely thus the entire engine flow passes the turbine. As plotted in Figure 2(b), the turbine flow resembles a nozzle model but it does not reach the choking point at pressure ratio about 0.53 as the nozzle model does [12, 18]. Therefore in this work the turbine flow is estimated using a single nozzle model but with an effective pressure ratio (Figure 3) to avoid early choking prediction.

The effective pressure ratio is calculated from the turbine actual pressure ratio using:

\[
Pr_{\text{eff}} = (P_{\text{ds}} / P_{\text{us}})^{\alpha}.
\]

Thus, the turbine flow is estimated by the following:

\[
\dot{m}_{\text{turb}} = \frac{P_{\text{us}}}{\sqrt{RT_{\text{us}}}} \cdot \frac{A_{\text{eff}} \cdot f(Pr_{\text{t}})}{} \tag{2-a}
\]

\[
f(Pr_{\text{t}}) = \frac{2e}{\sqrt{\gamma_{e} - 1}} \cdot Pr_{\text{t}}^{e/\gamma_{e} - 1} - Pr_{\text{t}}^{e/\gamma_{e} + 1} \tag{2-b}
\]

\[
Pr_{\text{t}} = \max(Pr_{\text{t}}, \left(\frac{2e}{\gamma_{e} + 1}\right)^{e/\gamma_{e} - 1}) \tag{2-c}
\]

\[
P_{\text{t}} = \frac{P_{\text{ds}}}{P_{\text{us}}} = \left(\frac{P_{\text{ds}}}{P_{\text{us}}}\right)^{\alpha} \tag{2-d}
\]

where \(A_{\text{eff}}\) is the effective area of the turbine and \(\alpha\) is a constant to include the turbine effective pressure ratio in the turbine flow model instead of its real pressure ratio. More details on how to optimally estimate \(\alpha\) and to calculate \(A_{\text{eff}}\) are presented in [19].

![Figure 2: (a) Typical engine speed-torque profile during the WG and turbine validation tests, (b) comparison between the turbine corrected flow and a conventional orifice corrected flow](image-url)
different measured WG positions. As shown, the discharge factor has a linear relation to the WG displacement. The linear relation between the WG displacement and the discharge factor makes it easy to estimate the WG flow using Equation 3 by the ECU. However, the WG position is not measured in some production TC engines and it needs to be estimated. In fact the routine practice is that the ECU reaches a desired set point for the turbocharger using an estimation of WG flow based on prediction of $X_{WG}$.

![Figure 3: The turbine schematic and the 0-D model; (a) real turbine and (b) 0-D model](image)

![Figure 4: Modeled turbine flow compared to measure engine flow with closed WG](image)

**4) Modeling the WG flow and displacement**

The WG is an orifice with a variable area. Therefore the flow over the wastegate is modeled using the orifice isentropic equation [14-15],

$$m_{WG} = AC_d \frac{P_{a,t}}{\sqrt{RT_{a,t}}} f \left( \frac{P_{a,t}}{P_{a,t}} \right)$$  \hspace{0.5cm} (3)

where $f$ comes from Equation 2-b and the discharge factor is a function of the WG actuator position, $AC_d = f(X_{WG})$. The WG flow in Equation 3 can also be calculated from the difference between the engine flow and the turbine flow,

$$\dot{m}_{WG} = \left( \dot{m}_{air} + \dot{m}_{fuel} \right) - \dot{m}_t$$  \hspace{0.5cm} (4)

where $\dot{m}_{air}$ is the engine air mass flow, $\dot{m}_{fuel}$ is the injected fuel mass flow and $\dot{m}_t$ is the turbine mass flow from Equation 2. It is assumed that the estimation of $\dot{m}_t$ from Equation 2 remains valid when the WG is open. When the WG flow is known from Equation 4, the discharge factor can be computed using Equation 3. Figure 5 shows $AC_d$ in

![Figure 5: WG discharge factor in different WG positions](image)

The WG position is predicted based on forces applied to its actuator and the flapper. Figure 6 shows the schematic of the wastegate and its force diagram. As illustrated, there are three main sources of force which affect the WG position. The first force is from the pressure difference inside the actuator capsule ($F_{cap}$) which opens the WG. This pressure difference is controlled by the electronic solenoid valve which regulates the capsule pressure by incorporating pressures downstream and upstream the compressor. The second source of the force is the WG spring force ($F_s$) preloaded by an initial displacement, $X_0$. The spring is to close the WG allowing more flow over the turbine. The third force is applied by the exhaust gas flow inside the exhaust manifold, $F_{flow}$. Therefore, when the WG is open, $X_{WG}$ is estimated using the torque balance at point A, shown in Figure 6, as:

$$[F_s - F_{cap}]r_2 \cos(\theta) - F_{flow}r_1 = \left[ K_{sp} \left( X_{WG} + X_0 \right) - (P_{us,t} - P_{ds,t}) A_{cap} \right] r_2 \cos(\theta)$$  \hspace{0.5cm} (5)

in which, $A_{cap}$ is the area of the diaphragm inside the capsule, $r_1$ and $r_2$ are the length of the connecting rods and $K_{sp}$ is the spring stiffness.

The flow force is calculated by applying the conservation of momentum for a control volume around the WG flapper (Figure 6) as:

$$F_{flow} + \int_{CV} T dA + \int_{CV} \beta p dV = \int_{CV} V (\rho \nu dA) + \frac{\partial}{\partial t} \int_{CV} V (\rho dV)$$  \hspace{0.5cm} (6)

where $T$ is the surface force in N/m², $\beta$ is the body force in N/kg, $\nu$ is the volume of an element inside the control volume in m³, $\rho$ is the exhaust gas density in
kg/m$^3$ and $V$ is the fluid velocity in m/sec. The following are assumptions used to calculate $F_{flow}$ from Equation 6:

- Since we present models to predict steady-state flow, the time dependency term is eliminated ($\frac{\partial}{\partial t} = 0$).

- The effect of body forces exerted by the surroundings are ignored ($\beta = 0$). The body force is mainly exerted by the gravity and the weight of elements inside the control volume is negligible.

- The pressure in the front of the flapper is $P_{int}$ and in the back of the flapper is $P_{out}$ when the WG opens. The assumption is considered for small displacement of the WG, where $X_{WG}$ affects $m_{WG}$ noticeably. This means the fluid pattern from the WG hole to the WG flapper is assumed as the pattern inside an orifice. In an orifice, a large portion of the pressure drop for a low viscous fluid happens at the orifice outlet section [22].

- The gas flow hitting the flapper surface leaves it parallel to the surface.

Therefore the flow force from Equation 6 is:

$$F_{flow} = [m_{WG} V_i + (P_{int} - P_{out}) A_{WG}] =$$

$$\left[\frac{R_{w} T_{w} m_{WG}^2 P_{WG}}{P_{int} A_{WG,b}} + (P_{int} - P_{out}) A_{WG}\right] \cos(\theta)$$

(7)

in which $A_{WG}$ and $A_{WG,b}$ are the cross sections of the flapper and the WG hole respectively (Figure 6). In Equation 7, $m_{WG} V_i$ is the force from the gas kinetic energy ($F_{flow,K}$) and $(P_{int} - P_{out}) A_{WG}$ is the force from the exhaust pressure, ($F_{flow,P}$). Plugging Equation 7 into Equation 5, the WG displacement from its equilibrium point is estimated as the following:

$$X_{WG} = \frac{F_{flow,K} r_2 + F_{flow,P} r_2}{K_{sp} r_2} - X_0 : F_{flow,K} r_2 + F_{flow,P} r_2 > F_{X,0} r_2 \quad (8-a)$$

$$X_{WG} = 0 : F_{flow,K} r_2 + F_{flow,P} r_2 < F_{X,0} r_2 \quad (8-b)$$

where $F_{X,0}$ is the WG initial load. As Equation 8 proposes, the capsule and flow forces should be large enough to cancel the WG spring force otherwise the WG stays in its initial position ($X_{WG} = 0$).

Figure 7 depicts results of WG modeling during a test procedure similar to Figure 2(a), but the WG operates normally in this new test and is not kept closed. In Figure 7(a), the modeled WG flow from Equation 3 with a known $X_{WG}$ is compared to the WG flow computed from Equation 4. In Figure 7(b) the WG position calculated from Equation 8 is compared to the measured WG position. Results of both WG flow and displacement models reveal good agreement between measured and estimated data.

![Figure 6: Wastegate schematic and its force diagram](image)

5) Compressor air flow model

The air flow through a centrifugal compressor can be predicted using its characteristic map once the compressor pressure ratio and rotational speed is known. Figure 8(a) shows the compressor characteristic map along with results of three different tests; a) a full mapping test of the engine b) a test in which the WG close-command is set to fully open (WGC = 0%) and, c) a test with the WG close-command set to fully-closed (WGC = 100%). As shown, the compressor characteristic map represents the compressor flow and pressure ratio at specific compressor rotational speeds. Therefore at a speed not included in the map, the compressor flow can be predicted interpolating in the map. However, linear interpolation in the characteristic map introduces large errors for the air mass flow prediction [9, 11]. Moreover the map is not suitable for analytical works like the stability investigation [16].

Dimensional analysis is applied to the compressor map to detect the compressor airflow reducing the interpolation error [23]. Two non-dimensional variables known as the head parameter, $\Psi$ and the normalized compressor flow rate, $\Phi$ are defined as the following:
The weak dependency of \( \Phi \) on \( \Psi \) and the width of transformed compressor map are two sources of error when the fitted curve is used for the compressor flow estimation.

In a turbocharged engine, the coupling between the compressor and the turbine created by the engine limits the range of the compressor operation in its characteristic map for a specific environmental pressure and temperature conditions (Figure 8(a)). As plotted, there is a one by one relation between the compressor flow and its pressure ratio before point \( \Phi = \Psi \) during the mapping test. In mass flows greater than that of point \( \Phi = \Psi \), engine may have different pressure ratios for each mass flow. The reason is that there are two options for the ECU to get to a specific operation point. This implies that the compressor operating point can be detected knowing the WG position and turbine flow.

The effect of the WG position on the turbocharger operation is investigated in two extreme test conditions. In the first test, the WG is left open as much as possible depending on the compressor downstream pressure, WGC = 0%, whereas in the second test the WG is fully closed, WGC=100%. Figure 8(a) depicts the operation line of the compressor in the two tests. As expected, all other possible working points are located between these two extreme operating lines.

Figure 8: (a) The compressor characteristic map and its trajectory during the engine test points, (b) Transformation results from the compressor map and the engine test points.

To use the effect of the WG position for prediction of the compressor working point, the modified compressor head parameter is defined as:

\[
\Psi_{\text{mod}} = \frac{\Psi f(P_{\text{ms}}, P_{\text{t}})}{f(P_{\text{t}})}
\]

where \( f(P_{\text{t}}) \) comes from Equation 2-b. As Equation 10 suggests, the \( \Psi_{\text{mod}} \) uses the exhaust manifold information using the turbine pressure ratio (either estimated [5] or measured) and corrected flow. The effect of modifying terms in Equation 10 is observed by plotting the compressor corrected air flow against \( \Psi \) and \( \Psi_{\text{mod}} \) in Figure 9(a) and Figure 9(b) respectively.
As illustrated, the compressor flow shows very weak sensitivity to \( \Psi \) which causes high error if it is used for air flow estimation. This problem is resolved by introducing \( \Psi_{\text{mod}} \). As Figure 9(b) suggests:

- The compressor flow is sensitive enough to the \( \Psi_{\text{mod}} \).
- Before the WG affecting forces get high enough to open it, all the compressor operating points are positioned on a unique line. Therefore, the compressor air flow is predicted in this region using a fitted curve to the test points of either WGC = 0% or WGC = 100% experiments.
- After the WG opens, two different but very close curves represent the relations between \( m_{\text{corr,c}} \) and \( \Psi_{\text{mod}} \). In this region, the compressor flow for other WGC values are predicted by interpolation between the two operation lines of WGC = 0% and WGC = 100%.

The proposed strategy to calculate \( m_{\text{corr,c}} \) is checked in two experiments with two different constant WG positions; one with WGC = 30% and the other with WGC = 60% (Figure 10). The results of the new model are compared to results from a linear interpolation inside the compressor characteristic map. As suggested, before the WG opens, the compressor airflow is estimated using either WGC = 0% or WGC = 100% maps in Figure 9(b).

When the WG opens a linear interpolation between the two curves in Figure 9(b) is used. The instant of WG opening is predicted using the WG displacement model in Equation 8.

As Figure 10 shows, there is good match between measured and modeled results for both tests. The average error, \( \bar{e} \), for the modeled compressor corrected flow in the experiment with WGC = 30% is 0.9% with a standard deviation of \( \sigma = 2.8\% \) and for the experiment with WGC = 60%; \( \bar{e} = 0.2\% \) with \( \sigma = 2.3\% \). For interpolated data, the WGC = 30% test has \( \bar{e} = -24\% \) with \( \sigma = 11\% \) and the WGC = 60% test has \( \bar{e} = -30\% \) with \( \sigma = 6.7\% \).

6) Conclusion

Steady state real-time models suitable for engine control are presented to estimate turbocharger working conditions. The turbine flow is estimated using the model of an isentropic nozzle with an effective pressure ratio. The effective pressure is calculated from the turbine actual pressure ratio to avoid prediction of early choking for the turbine. The WG flow is predicted using the orifice model for compressible fluids, if the WG position is measured. For engines that the WG position is not measured, its position is estimated using a physically sensible model of the WG and its affecting forces. Experimentally validated WG displacement model takes into account forces form the WG passing flow, capsule pressure, and spring.

Another model for prediction of flow over the compressor is proposed, which utilizes the turbine corrected flow and effective pressure ratio to modify the compressor head parameter. The modified compressor head parameter is found in good correlation with the compressor mass flow and is used to predict it. The predicted compressor flow by the new model matches well with the experimentally measured flow over the compressor.
Nomenclature

\begin{align*}
A & \quad \text{Area (m}^2\text{)} \\
C_d & \quad \text{Discharge factor (kg/m}^2\text{)} \\
C_p & \quad \text{Specific heat at constant pressure (kJ/kg K)} \\
d & \quad \text{Diameter (m)} \\
F & \quad \text{Force (N)} \\
K & \quad \text{Spring stiffness (N/m)} \\
im & \quad \text{Mass flow (kg/s)} \\
P & \quad \text{Pressure (kPa)} \\
P_r & \quad \text{Pressure ratio (-)} \\
R & \quad \text{Gas constant (kJ/kg K)} \\
T & \quad \text{Temperature (K)} \\
U & \quad \text{Speed (m/s)} \\
X & \quad \text{Displacement (m)} \\
\gamma & \quad \text{Ratio of specific heats (-)} \\
\rho & \quad \text{Density (kg/m}^3\text{)} \\
\omega & \quad \text{Rotational speed (rad/s)}
\end{align*}

Subscripts

\begin{align*}
a & \quad \text{air} \\
b & \quad \text{boost} \\
c & \quad \text{compressor} \\
cap & \quad \text{capsule} \\
corr & \quad \text{corrected} \\
d & \quad \text{downstream} \\
e & \quad \text{exhaust} \\
eff & \quad \text{effective} \\
h & \quad \text{hole} \\
im & \quad \text{intake manifold} \\
K & \quad \text{kinetic} \\
P & \quad \text{pressure} \\
\text{ref} & \quad \text{reference} \\
sp & \quad \text{spring} \\
t & \quad \text{turbine} \\
us & \quad \text{upstream} \\
WG & \quad \text{wastegate}
\end{align*}

References


الگونمایی پیاپی پرخوران در موتورهای اشتعال جرقه‌ای

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چکیده
در برنامه‌ریزی رايانه موتورهای اشتعال جرقه‌ای که به روش ساختار کانتور طراحی شده‌اند، تخمین جریان‌هایی از محدوده‌های مختلف و سیار مهم می‌باشد. در این مقاله، روابطی برای تخمین شار عبوری از گرداب درجه‌های هدررو و تنجار، به صورت مقدار میانگین در یک موتور احتراق داخلی مرسی می‌باشد که دارای حسگر جریان هوای داخلی است. اثرات گردابه‌ای که از گرداب افتاده گردیده است، جریان عبوری از گرداب با استفاده از الگوی جریان اوریفسی و نسبت توده به قطر گرداب، انرژی نسبت به نشان‌دهنده نسبت به نقطه مقابل جریان عبوری از گرداب به شکلی ساده و ساده می‌باشد. در نهایت، برای تخمین جریان عبوری از گرداب از تجربه‌های زده شده استفاده می‌شود. در این مقاله، از تجربه‌های انجام‌شده در موتور پنلی، از تجربه‌های انجام‌شده در موتور پنلی بهره می‌برند. نتایج آزمون‌های انجام‌شده نشان می‌دهد که تمامی مقالات موتور ایران محفوظ است.