Steady-state modeling of a turbocharger in gasoline engines

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ABSTRACT

The gas flow estimation is crucial for the proper operation and monitoring of turbocharged (TC) engines with a torque structured engine control unit (ECU). This paper presents mean value models developed for modeling gas flow over the compressor, the turbine and the wastegate (WG) in a TC gasoline engine not equipped with a hot film air-mass flow meter (HFM). The turbine flow was modeled by an isentropic nozzle with an effective pressure ratio. A physically sensible model was developed for estimating the WG flow and its position which estimates them over a wide range of the engine operating conditions. Finally, a novel model for estimating the compressor air flow was proposed, using coupling between powers generated by the turbine and used by the compressor. All models are simple and accurate enough to be utilized for engine real-time control applications. Experimental results from a modern gasoline TC engine were used to validate all the models. Prediction results from models were in very good agreement with the experimental measurements.

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1) Introduction

Engine downsizing using turbochargers in gasoline engines is a common solution to reducing fuel consumption and emission for automotive manufacturers [1]. In fact, displacement reduced turbocharged engines regenerate thermal power from exhaust gas either by a wastegate controlled or a variable geometry turbine. The turbochargers controlled by the wastegate (WG) benefit low cost and high durability in the harsh thermal environment of the exhaust manifold in the current automotive market [2].

When a turbocharger is added to a naturally aspirated engine, different strategies should be included in the engine control unit (ECU). These strategies are either to control the turbocharger [3-4] or to monitor its performance [5] for proper and safe operation of both the engine and turbocharger. Among the strategies, gas flow estimation is imperative for the ECU to control the gasoline engine torque and emission. The ECU requires either measuring or estimating the air mass flow as it enters the intake path from the air filter and continues monitoring the flow stream line until it exits from the tailpipe. Mean-Value Modeling (MVM) is the main approach implemented in a production ECU for flow estimations. The MVM concept considers the engine parameters as constants during a four-stroke engine cycle and neglects cyclic variation of the parameters. First order ordinary differential equations are the major relations to describe the engine dynamics. Simple structure of MVM is shown to be accurate enough even for transient engine control applications [6-8].

For engines not equipped with an HFM, the air flow is neither measured nor estimated before it reaches the throttle. However, estimation of the air mass flow in the path from the compressor to the throttle is important particularly for monitoring purposes such as intercooler efficiency and air leak detection in the path. The compressor flow can be predicted using its characteristic map [9], but with large errors from linear interpolation and extrapolation [10-11]. In the other side of the engine, there exist challenges with estimation of gas flow over the turbine and the WG when the WG is not closed with mean value solutions such as [12-15].

This paper presents MVM formulations to estimate the wastegate, turbine and compressor flows suitable for real-time engine control applications. The turbine flow is estimated from an orifice isentropic compressible flow model with an effective pressure ratio calculated with a new approach from the turbine actual pressure ratio. Then, the turbine flow model is used to estimate parameters required to develop a physically sensible model for the wastegate position and flow. As a new method, the coupling between the compressor and the turbine is used to predict the compressor flow which can be calibrated on a engine test bench easily. The compressor flow model can be replaced by existing models to improve the accuracy of estimation algorithms presented in [9, 16]. The proposed models for the turbocharger main components are easy to be calibrated on a new engine and developed such that their real-time application in the ECU sounds sensible when their input variables are available by measurements.

After the introduction, the experimental setup used for validation in all parts of this paper is described in section 2. Section 3 addresses modeling of the turbine gas flow. The WG model to estimate its flow and position in different engine operating points is explained in section 4. Finally the model for detection of the compressor air flow is outlined in section 5.

2) Experimental setup

A 4-cylinder port fuel injected gasoline turbocharged engine is used for validation of all models in this work. As shown in Figure 1, the engine is equipped to a turbocharger with a radial turbine and a centrifugal compressor.

The turbine is controlled by a wastegate which allows bypassing the exhaust gas from the turbine. Opening the WG reduces exhaust back pressure (which improves the fuel economy) in the operation points where the engine does not require high boost pressure. The WG position is controlled by a pneumatic actuator. The pressure inside the actuator is controlled by an electronic solenoid valve which combines pressures downstream and upstream the compressor. Figure 1 also illustrates the engine measurement setup instrumented to a range of different sensors. A proximity sensor is used to measure the rotor speed with ±500 rpm resolution. Temperatures are measured using K-type thermocouples with an accuracy of ±2 °C and pressures are measured using piezoresistive pressure transmitters with ±0.1 kPa accuracy. The fuel mass flow is measured using a temperature-controlled fuel mass flow meter with a measurement accuracy of 0.12% and maximal measuring frequency of 20 Hz. The measured fuel in the test cell is used along with the measured engine air-fuel ratio...
to compute the air-charge required to validate flow models in this work since the engine is not equipped with an HFM. A hall-effect sensor with -40 to +150°C working temperature is integrated to the engine setup for measuring the WG displacement. The hall sensor is connected to the WG actuator using a thermal isolating pad. Further specifications of the engine are presented in Table 1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>1.7</td>
<td>lit</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>9.9</td>
<td>-</td>
</tr>
<tr>
<td>No. of cylinders</td>
<td>4</td>
<td>-</td>
</tr>
<tr>
<td>Bore × Stroke</td>
<td>78.6 × 85</td>
<td>mm</td>
</tr>
<tr>
<td>Max. torque</td>
<td>215 @ 2200-4800 rpm</td>
<td>N.m</td>
</tr>
<tr>
<td>Max. power</td>
<td>110 @ 5500 rpm</td>
<td>kW</td>
</tr>
<tr>
<td>Max. accessible WG Disp.</td>
<td>12</td>
<td>mm</td>
</tr>
</tbody>
</table>

Thus, the turbine flow is estimated by the following:

\[ \dot{m}_t = \frac{P_{a_{ref}}}{\sqrt{RT_{a_{ref}}}} \cdot A_{eff} \cdot f(P_{r_{t}}^*) \]  

\[ f(P_{r_{t}}^*) = \left( \frac{2 \gamma'_{r}}{\gamma'_{r} - 1} \right) \left( P_{r_{t}}^{2/3} - P_{r_{t}}^{1/3} \right) \]  

\[ P_{r_{t}}^* = \max \left( P_{r_{t}} \left( \frac{2}{\gamma'_{r} - 1} \right) \right) \]  

\[ P_{r_{t}} = \left( \frac{P_{d_{ref}}}{P_{a_{ref}}} \right)^{\alpha} \]  

where \( A_{eff} \) is the effective area of the turbine and \( \alpha \) is a constant to include the turbine effective pressure ratio in the turbine flow model instead of its real pressure ratio. More details on how to optimally estimate \( \alpha \) and to calculate \( A_{eff} \) are presented in [19].

3) Turbine flow model

Turbomachine performance characteristics such as mass flow and efficiency are specified by terms of pressure ratio and rotating speed. For these machines, the effect of inlet conditions are canceled by defining corrected mass flow and corrected speed. Therefore, for a machine of a specified size which handles a single gas, the dimensional analysis to compressible fluids suggests the following [17]:

\[ \dot{m}_{corr} = f(P_{r}, \omega_{corr}) \]  

(1)

where \( \omega_{corr} = \omega \sqrt{T_{us}} \) is the corrected rotating speed of the machine shaft and the corrected mass flow is defined as \( \dot{m}_{corr} = \dot{m} \sqrt{T_{us}} / P_{a_{us}} \) for a turbine and \( \dot{m}_{corr} = \dot{m} \sqrt{T_{ds}} / P_{a_{ds}} \) for a compressor. The pressure ratio, \( P_{r} \), can be either the upstream to downstream turbomachine pressure ratio or the reverse.

Radial turbines have rather weak dependency upon the corrected speed [17]; and the pressure ratio term is the most significant variable in Equation 1. Figure 2 compares the measured corrected flow over the radial turbine with that of a conventional nozzle with a unit discharge area. The test procedure for measuring the presented data is illustrated in Figure 2(a). The flow over the turbine is measured by closing the wastegate completely thus the entire engine flow passes the turbine. As plotted in Figure 2(b), the turbine flow resembles a nozzle model but it does not reach the choking point at pressure ratio about 0.53 as the nozzle model does [12, 18]. Therefore in this work the turbine flow is estimated using a single nozzle model but with an effective pressure ratio (Figure 3) to avoid early choking prediction.

The effective pressure ratio is calculated from the turbine actual pressure ratio using:

\[ P_{r_{eff}} = \left( \frac{P_{d_{ref}}}{P_{a_{ref}}} \right)^{\alpha} \]  

Figure 2: (a) Typical engine speed-torque profile during the WG and turbine validation tests, (b) comparison between the turbine corrected flow and a conventional orifice corrected flow.

The turbine flow modeled from Equation 2 is compared to the measured engine flow in Figure 4. In Equation 2, the turbine upstream and downstream pressure and also upstream temperature are measured however, they can be estimated as addressed in [19-20]. As shown, predicted flow for the turbine agrees well with measured gas flow. The estimation error of the results in Figure 4 has an average of 1.4% with a standard deviation, \( \sigma \), of 1.5.
different measured WG positions. As shown, the discharge factor has a linear relation to the WG displacement. The linear relation between the WG displacement and the discharge factor makes it easy to estimate the WG flow using Equation 3 by the ECU. However, the WG position is not measured in some production TC engines and it needs to be estimated. In fact the routine practice is that the ECU reaches a desired set point for the turbocharger using an estimation of WG flow based on prediction of $X_{WG}$.

The WG position is predicted based on forces applied to its actuator and the flapper. Figure 6 shows the schematic of the wastegate and its force diagram. As illustrated, there are three main sources of force which affect the WG position. The first force is from the pressure difference downstream and upstream the compressor. The second source of the force is the WG spring force ($F_{sp}$) preloaded by an initial displacement, $x_0$. The spring is to close the WG allowing more flow over the turbine. The third force is applied by the exhaust gas flow inside the exhaust manifold, $F_{flow}$. Therefore, when the WG is open, $X_{WG}$ is estimated using the torque balance at point A, shown in Figure 6, as: $$F_{flow} - F_{cap} - \beta \cdot \cos(\theta) - F_{sp} = 0$$

(5)

in which, $F_{cap}$ is the area of the diaphragm inside the capsule, $r_1$ and $r_2$ are the length of the connecting rods and $K_{cap}$ is the spring stiffness. The flow force is calculated by applying the conservation of momentum for a control volume around the WG flapper (Figure 6) as:

$$F_{flow} = \oint_{CV} T_{dA} + \oint_{CS} \beta \cdot \rho \cdot V \cdot dV$$

(6)

where $T$ is the surface force in N/m², $\beta$ is the body force in N/kg, $\rho$ is the volume of an element inside the control volume in m³, $\rho$ is the exhaust gas density in
kg/m³ and \( V \) is the fluid velocity in m/sec. The
following are assumptions used to calculate \( F_\text{flow} \) from
Equation 6:

- Since we present models to predict steady-state
  flow, the time dependency term is eliminated
  \( (\partial / \partial t = 0) \).
- The effect of body forces exerted by the
  surroundings are ignored \( (\beta = 0) \). The body force
  is mainly exerted by the gravity and the weight of
  elements inside the control volume is negligible.
- The pressure in the front of the flapper is \( P_{\text{act}} \) and
  in the back of the flapper is \( P_{\text{dst}} \) when the WG
  opens. The assumption is considered for small
  displacement of the WG, where \( X_{\text{WG}} \) affects
  \( m_{\text{WG}} \) noticeably. This means the fluid pattern from
  the WG hole to the WG flapper is assumed as the
  pattern inside an orifice. In an orifice, a large
  portion of the pressure drop for a low viscous fluid happens at the orifice outlet section [22].
- The gas flow hitting the flapper surface leaves it
  parallel to the surface.

Therefore the flow force from Equation 6 is:

\[
F_\text{flow} = \left[ m_{\text{WG}} V_1 + (P_{\text{act}} - P_{\text{dst}}) A_{\text{WG}} \right] \frac{R_{\text{m,WG}} T_{\text{m,WG}} P_{\text{WG}}^2}{P_{\text{m,WG,b}}} (P_{\text{act}} - P_{\text{dst}}) A_{\text{WG}} \cos(\theta)
\]

in which \( A_{\text{WG}} \) and \( A_{\text{WG,b}} \) are the cross sections of the flapper and the WG hole respectively (Figure 6). In
Equation 7, \( m_{\text{WG}} V_1 \) is the force from the gas kinetic energy \( (F_{\text{flow,k})} \) and \( (P_{\text{act}} - P_{\text{dst}}) A_{\text{WG}} \) is the force from
the exhaust pressure, \( (F_{\text{flow,p})} \). Plugging Equation 7
into Equation 5, the WG displacement from its
equilibrium point is estimated as the following:

\[
X_{\text{WG}} = \frac{F_{\text{cap}} r_1 + F_{\text{cap}} r_2}{K_{\text{cap}}} - X_0 : F_{\text{cap}} r_2 + F_{\text{cap}} r_1 > F_{x,0} r_2 \quad (8-a)
\]

\[
X_{\text{WG}} = 0 : F_{\text{cap}} r_2 + F_{\text{cap}} r_1 < F_{x,0} r_2 \quad (8-b)
\]

where \( F_{x,0} \) is the WG initial load. As Equation 8
proposes, the capsule and flow forces should be large
enough to cancel the WG spring force otherwise the
WG stays in its initial position \( (X_{\text{WG}} = 0) \).

Equation 7 depicts results of WG modeling during a test
procedure similar to Figure 2(a), but the WG operates
normally in this new test and is not kept closed. In
Figure 7(a), the modeled WG flow from Equation 3
with a known \( X_{\text{WG}} \) is compared to the WG flow
computed from Equation 4. In Figure 7(b) the WG
position calculated from Equation 8 is compared to
the measured WG position. Results of the both WG
flow and displacement models reveal good agreement
between measured and estimated data.

Figure 7: Results of waste gate model (a) waste gate flow
and (b) waste gate displacement

5) Compressor air flow model

The air flow through a centrifugal compressor can be
predicted using its characteristic map once the
compressor pressure ratio and rotational speed is
known. Figure 8(a) shows the compressor
characteristic map along with results of three
different tests; a) a full mapping test of the engine b)
a test in which the WG close-command is set to fully
open (WG = 0%) and, c) a test with the WG close-
command set to fully-closed (WG = 100%). As
shown, the compressor characteristic map represents
the compressor flow and pressure ratio at
specific compressor rotational speeds. Therefore at a
speed not included in the map, the compressor flow
can be predicted interpolating in the map. However,
linear interpolation in the characteristic map
introduces large errors for the air mass flow
prediction [9, 11]. Moreover the map is not suitable
for analytical works like the stability investigation
[16].

Dimensional analysis is applied to the compressor
map to detect the compressor airflow reducing the
interpolation error [23]. Two non-dimensional
variables known as the head parameter, \( \Psi \) and the
normalized compressor flow rate, \( \Phi \) are defined as the
following:

**Figure 6: Wastegate schematic and its force diagram**
operation point. This implies that the compressor operating point can be detected knowing the WG position and turbine flow.

The effect of the WG position on the turbocharger operation is investigated in two extreme test conditions. In the first test, the WG is left open as much as possible depending on the compressor downstream pressure, WGC = 0%, whereas in the second test the WG is fully closed, WGC=100%. Figure 8(a) depicts the operation line of the compressor in the two tests. As expected, all other possible working points are located between these two extreme operating lines.

\[
\Psi = \frac{C_p, a T_{US}, c}{0.5U_f^2} \left[ \left( \frac{P_{ds,c}}{P_{US,c}} \right)^{(\gamma-1)/\gamma} - 1 \right] \\
U_f = \pi / 60 \cdot d_i \cdot \omega_i \\
\Phi = \dot{m}_i / (\rho_{ac} \cdot d_i^2 \cdot \omega_i)
\]

where \(d_i\) is the compressor impeller diameter in \(m\), \(U_f\) is the impeller tip velocity in \(m/s\) and \(\rho_{ac}\) is the air density in \(kg/m^3\). The result of this transformation is shown in Figure 8(b). As it can be observed, the wide and scattered operating points in a compressor characteristic map are transformed into a band by the \(\Phi-\Psi\) transformation. The band is represented by a curve fitted to the transformation results (Figure 8(b)). Then the curve can be used for prediction of the compressor working point. Figure 9 also shows the compressor working points (Figure 8(a)) and their transformation results (Figure 8(b)) during an engine mapping test which covers most of the engine operating points. As plotted, when the compressor is coupled to the engine, \(\Phi-\Psi\) relation change to a specific trajectory with weak dependency of \(\Phi\) on \(\Psi\) particularly in part loads of the engine (can be represented by a rough estimation as \(\Phi<0.01\)). The weak dependency of \(\Phi\) on \(\Psi\) and the width of transformed compressor map are two sources of error when the fitted curve is used for the compressor flow estimation.

In a turbocharged engine, the coupling between the compressor and the turbine created by the engine limits the range of the compressor operation in its characteristic map for a specific environmental pressure and temperature conditions (Figure 8(a)). As plotted, there is a one by one relation between the compressor flow and its pressure ratio before point "A" during the mapping test. In mass flows greater than that of point "A", engine may have different pressure ratios for each mass flow. The reason is that there are two options for the ECU to get to a specific \(m_{corr,c}\) in the part load points in each engine speed. One option is opening the engine air throttle and setting the WGC = 0% which leaves the WG open using the entire downstream compressor pressure. This option has the advantage of improving the engine fuel consumption since it maximizes the WG flow and reduces the exhaust back pressure. The second option is keeping the throttle constant and closing the wastegate instead (WGC # 0%). The test with closed wastegate resembles "sport calibration" when fast response of the engine to the driver commands is the target.

The turbine and its pressure ratio are also affected within the two engine modes. In the fuel economy mode (WGC = 0%) the turbine pressure ratio and passing flow is lower than that of in the sport driving mode (WGC # 0%). Therefore the wastegate position (or the ECU command to close the WG WGC) is a key variable which changes the compressor and turbine

To use the effect of the WG position for prediction of the compressor working point, the modified compressor head parameter is defined as:

\[
\Psi_{mod} = \frac{\Psi^* (P_{ds,corr} P_{c,tur} )^a}{f(P_t^*)}
\]

where \(f(P_t^*)\) comes from Equation 2-b. As Equation 10 suggests, the \(\Psi_{mod}\) uses the exhaust manifold information using the turbine pressure ratio (either estimated [5] or measured) and corrected flow. The effect of modifying terms in Equation 10 is observed by plotting the compressor corrected air flow against \(\Psi\) and \(\Psi_{mod}\) in Figure 9(a) and Figure 9(b) respectively.
As illustrated, the compressor flow shows very weak sensitivity to \( \psi \) which causes high error if it is used for air flow estimation. This problem is resolved by introducing \( \psi_{mod} \). As Figure 9(b) suggests:

- The compressor flow is sensitive enough to the \( \psi_{mod} \).
- Before the WG affecting forces get high enough to open it, all the compressor operating points are positioned on a unique line. Therefore, the compressor air flow is predicted in this region using a fitted curve to the test points of either WGC = 0% or WGC = 100% experiments.
- After the WG opens, two different but very close curves represent the relations between \( m_{corr,c} \) and \( \psi_{mod} \). In this region, the compressor flow for other WGC values are predicted by interpolation between the two operation lines of WGC = 0% and WGC = 100%.

The proposed strategy to calculate \( m_{corr,c} \) is checked in two experiments with two different constant WG positions; one with WGC = 30% and the other with WGC = 60% (Figure 10). The results of the new model are compared to results from a linear interpolation inside the compressor characteristic map. As suggested, before the WG opens, the compressor airflow is estimated using either WGC = 0% or WGC = 100% maps in Figure 9(b).

![Figure 9: Compressor corrected flow as a function of \( \psi \) and modified \( \psi \)](image)

![Figure 10: Modeled and interpolated compressor air flow at different WG closing commands](image)

When the WG opens a linear interpolation between the two curves in Figure 9(b) is used. The instant of WG opening is predicted using the WG displacement model in Equation 8.

As Figure 10 shows, there is good match between measured and modeled results for both tests. The average error, \( \bar{e} \), for the modeled compressor corrected flow in the experiment with WGC = 30% is 0.9% with a standard deviation of \( \sigma = 2.8\% \) and for the experiment with WGC = 60%; \( \bar{e} = 0.2\% \) with \( \sigma = 2.3\% \). For interpolated data, the WGC = 30% test has \( \bar{e} = -24\% \) with \( \sigma = 11\% \) and the WGC = 60% test has \( \bar{e} = -30\% \) with \( \sigma = 6.7\% \).

6) Conclusion

Steady state real-time models suitable for engine control are presented to estimate turbocharger working conditions. The turbine flow is estimated using the model of an isentropic nozzle with an effective pressure ratio. The effective pressure is calculated from the turbine actual pressure ratio to avoid prediction of early choking for the turbine. The WG flow is predicted using the orifice model for compressible fluids, if the WG position is measured. For engines that the WG position is not measured, its position is estimated using a physically sensible model of the WG and its affecting forces. Experimentally validated WG displacement model takes into account forces form the WG passing flow, capsule pressure, and spring.

Another model for prediction of flow over the compressor is proposed, which utilizes the turbine corrected flow and effective pressure ratio to modify the compressor head parameter. The modified compressor head parameter is found in good correlation with the compressor mass flow and is used to predict it. The predicted compressor flow by the new model matches well with the experimentally measured flow over the compressor.
Nomenclature

\[ A \] Area (m²)  
\[ C_d \] Discharge factor (kg/m²)  
\[ C_p \] Specific heat at constant pressure (kJ/kgK)  
\[ d \] Diameter (m)  
\[ F \] Force (N)  
\[ K \] Spring stiffness (N/m)  
\[ m \] Mass flow (kg/s)  
\[ P \] Pressure (kPa)  
\[ P_t \] Pressure ratio (-)  
\[ R \] Gas constant (kJ/kg.K)  
\[ T \] Temperature (K)  
\[ U \] Speed (m/s)  
\[ X \] Displacement (m)  
\[ \gamma \] Ratio of specific heats (-)  
\[ \rho \] Density (kg/m³)  
\[ \omega \] Rotational speed (rad/s)

Subscripts

\( a \) air  
\( amb \) ambient  
\( b \) boost  
\( c \) compressor  
\( cap \) capsule  
\( corr \) corrected  
\( ds \) downstream  
\( e \) exhaust  
\( eff \) effective  
\( h \) hole  
\( im \) intake manifold  
\( K \) kinetic  
\( P \) pressure  
\( ref \) reference  
\( sp \) spring  
\( t \) turbine  
\( us \) upstream  
\( WG \) wastegate

References

الگونمایی پیاپی برخوران در موتورهای اشتعال جرقوی

چکیده
در برنامه رایانه موتورهای اشتعال جرقوی که به روش ساختار کشتی‌واپور طراحی شده‌اند، تخمین جریان عبوری از گردا، دریچه هدرروا و تنجار، به صورت مقدار میانگین در یک موتور احتراق داخلی بنزینی که دارای حسگر جریان عمودی از گردا و یک حسگر جریان اورفیسی آنتروپی ثابت و یک حسگر جریان عمودی از دریچه هدررا و یا میزان گشودگی آن ارائه گردیده است. در نهایت، با استفاده از اتصال بین گردا و تنجار در موتور اشتعال جرقوی، شرایط عبوری از تنگر تخمین زده شده است. در همه این سه حسگر جریان، با استفاده از روش‌های اندازه‌گیری به همراه با میزان گشودگی آن، داده‌های اندازه‌گیری شده از موتور بنزینی چهار استوانه‌ای آزمون‌آوری شدند. مقایسه نتایج داده‌های ارائه‌گری در این مقاله به منظور پیش‌بینی میزان گشودگی حسگر جریان تكنیکی مشترک شده است.

اطلاعات مقاله
نگهدارنده مقاله:
اولین انتشار: 87 خرداد 1396
پذیرش: 22 خرداد 1396
کلمات کلیدی:
برخوران
دریچه هدررو
تخمین جریان
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